MATH 647 – Spring 01

Homework 6 Due 03/23/01

1. Show that if we try to solve the heat equation with boundary conditions $u(0,t) = 0$, $u_x(l,t) = 0$ via separation of variables, we are led to the eigenfunctions

$$\sin(\pi x/2l),\ \sin(3\pi x/2l),\ \sin(5\pi x/2l),\ \ldots$$

.

2. Solve the following problem for the wave equation:

$$u_{tt} = u_{xx}, \qquad 0 < x < \pi$$
$$u(0,t) = u(\pi,t) = 0$$
$$u(x,0) = \sin(2x), \quad u_t(x,0) = 0,$$

using the method of separation of variables.

3. Problem 8, page 108. (Note that what the book calls $U(x)$ is the steady state solution that we have been calling $S(x)$.)

4. Solve the problem

$$u_t = u_{xx} + t\sin(2x), \qquad 0 < x < \pi$$
$$u(0,t) = u(\pi,t) = 0$$
$$u(x,0) = \frac{1}{16}\sin(2x),$$

as follows. Expand the solution $u(x,t)$ in the form

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) X_n(x),$$

where the X_n are the eigenfunctions of $-X'' = \lambda X$ with homogeneous Dirichlet boundary conditions and the $u_n(t)$ are to be determined. Using the inhomogeneous heat equation that we are trying to solve, arrive to ordinary differential equations for the $u_n(t)$ and solve them using the values of $u_n(0)$ given by the initial condition $u(x,0) = \frac{1}{16}\sin(2x)$. (Hint: the solution of $u_2' = -4u_2 + t$ is of the form $u_2(t) = ce^{-4t} + at + b$; you need to determine the constants.)